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Universal power law in the noise from a crumpled elastic sheet

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Using high-resolution digital recordings, we study the crackling sound that is emitted from crumpled sheets of Mylar as they are strained. These sheets possess many of the qualitative features of traditional disordered systems, including frustration and discrete memory. The sound can be resolved into discrete clicks, which are emitted during rapid changes in the rough conformation of the sheet. Observed click energies range over six orders of magnitude. The measured energy autocorrelation function for the sound is consistent with a stretched exponential $C(t) \sim \exp(-ct^{\beta})$, with $\beta \approx 0.35$. The probability distribution of click energies has a power law regime $p(E) \sim E^{-\alpha}$, where $\alpha \approx 1$. We find the same power law for a variety of sheet sizes and materials, suggesting that this $p(E)$ is universal.

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Physicists and laymen alike are familiar with the loud crackling noise produced when a crumpled cellophane candy wrapper is unwrapped. This noise is a generic product of thin, crumpled elastic sheets and is qualitatively different from the sounds that can be produced with a flat sheet. Despite its ubiquity and accessibility, to our knowledge there has been no experimental investigation of this sound. In this paper we present an elementary discussion of the generation of the sound and its dependence on the properties of the crumpled sheet. We develop a method to generate the sound which yields robust and reproducible results, and we make a quantitative study of the sound properties using high-resolution digital recordings.

The key distinction between flat and crumpled sheets is the quenched curvature disorder produced during the crumpling process. At high compressions some regions of the material become sharply creased [1], and the material at the fold flows plastically to relax the strain. The creases thus initiate linelike defects where the sheet prefers a nonzero curvature. If the sheet is subsequently uncrumpled it exhibits a rough texture due to the presence of these defects, which appear as a network of ridges in the sheet. For this reason we expect that a systematic investigation of the sound produced by these sheets will be relevant to a variety of issues in the theory of crumpled membranes. Thin crumpled sheets of paper and aluminum foil have been studied informally by several authors [2], and the theory of elastic sheets with quenched disorder is an active field of research [3]. Our research is also closely related to the work of Houle and Sethna, who study the sound produced when a flat sheet is crumpled [4].

Our first task was to make elastic sheets with quenched disorder in a uniform and reproducible way. We start with a flat sheet of Mylar [5], and successively crumple and uncrumple the sheet by hand. The result after one crushing is unsatisfactory for our purposes, since there are long-range correlations in the ridge network and we have doubts about the reproducibility of the result [2]. However, after thirty crumplings there is a dense network of overlapping ridges and an additional crumpling produces no obvious change in the appearance or the sound of the sheet. The final state is homogeneously rough down to a lower length scale ξ, which is limited by our ability to crush the sheet. If we estimate ξ for our samples using the size of the largest flat facets, we find $\xi \approx 100h$, where h is the thickness of the sheet.

The disordered sheets produced in this way have an unusual property. Manipulating a sheet by hand, we can find many inequivalent stable configurations. Figure 1 illustrates three configurations for the *same* sheet. These configurations can hold their shape for at least months. The reason for this behavior is the quenched curvature disorder. Figure 2 represents the crossing in the sheet of two ridges from two distinct crumpling events. Each ridge has a preferred dihedral angle $D \neq \pi$, but only one can be achieved. The energy of the pair is thus explicitly frustrated [6]. A sheet of linear size L will have $O(L^2/\xi^2)$ crossings. In this way the elastic energy of the sheet resembles the free energy of a spin glass. This is in accord with the suggestions of several investigators that membranes with quenched curvature disorder should exhibit glassy behavior at low temperature [3]. The multitude of stable configurations is a clear demonstration that the elastic energy of the sheet has many distinct minima. Typical energy barriers between minima are much greater than k_BT for a macroscopic sheet at room temperature.

Our recordings were made using an AKG Acoustics C34 microphone calibrated to have a linear response for frequencies from 20 Hz to 20 kHz. The analog signal from the microphone was converted to a 16-bit digital signal at a 48 kHz sampling rate by a Yamaha ProMix 01 mixer and recorded to digital audio tape. The recordings were made in a music studio that was neither soundproof nor anechoic. Since ambient room noise was only significant at frequencies below 200 Hz, we used the mixer as a high-

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pass filter with an attenuation of 15 dB below 100 Hz and no attenuation above 400 Hz. Most of the energy in our signal was above 1 kHz, so this was an adequate solution. To overcome the problem of echoes from the studio walls we constructed a soundbox by covering the inside of a cubical cardboard box 20 in. on a side with 3 in. of dense foam rubber packing material. One wall of the box was left open so that we could hold the crumpled sheets inside. We verified that the soundbox decreased the measured amplitude of echoes to negligible levels.

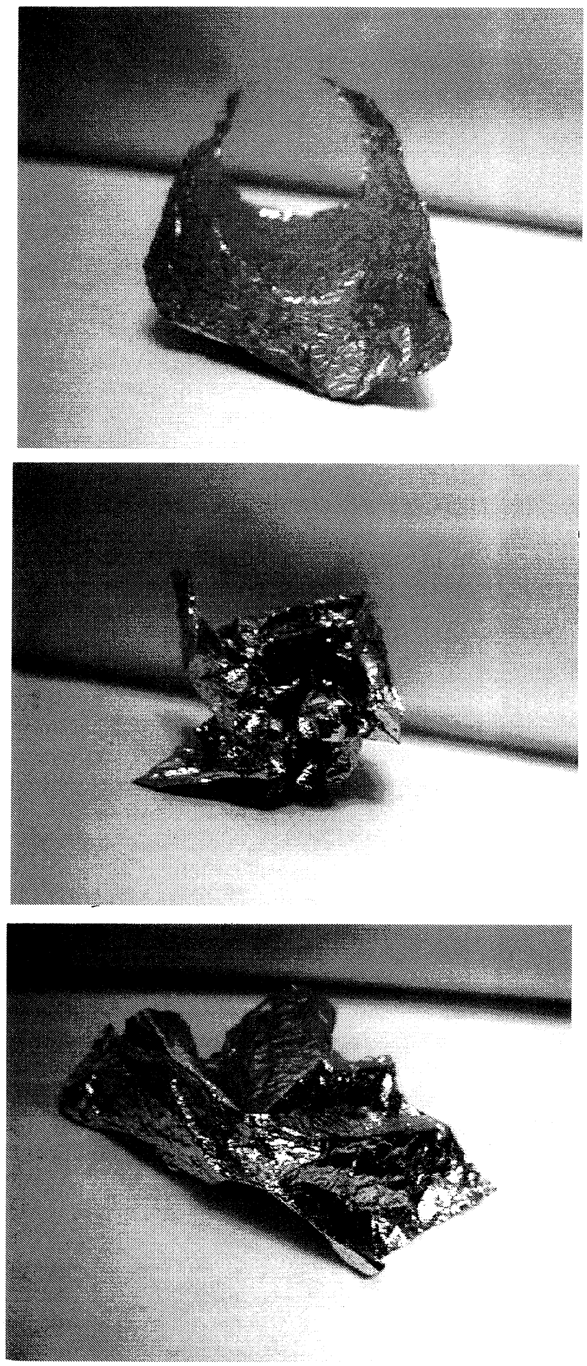

FIG. 1. Three stable conformations of a Mylar sheet with quenched curvature disorder. The dimensions of the sheet are 8 cm×9 cm×0.5 mil.

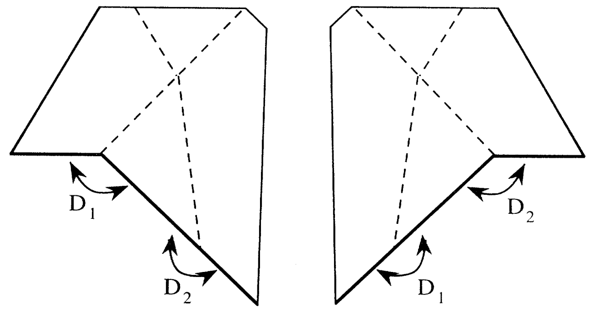

FIG. 2. Schematic illustration showing that one of two crossed ridges must have $D = \pi$.

The results presented here were recorded from a set of five square Mylar sheets prepared in a rough state as described above. The flat sheets were initially 10 cm on a side and 0.5, 1.0, 3.0, 5.0, and 10.0 mil thick, respectively. We produced the sound by holding a sheet by its edges in an approximately flat configuration and slowly twisting and shearing it in a cyclical fashion. Typical shearing rates were order 0.5 cm/sec. It was important to keep the sheet approximately flat to avoid two complications. (1) If the sheet rubbed against itself, it produced low-amplitude friction noise that diminished the sensitivity of our recordings. (2) If the sheet crumpled significantly, we would need to account for the attenuation of sound from the inner layers. The sheets were held about 20 cm from the microphone and each run was 10 sec long. We made ten runs in all, with each investigator taking a turn straining each sheet. To get the best use of the full energy scale, the gain of the mixer was changed for each run. We also made a 3-sec run for all five sheets with the gain fixed, so that we could compare the results from different sheets.

Figure 3 shows 1 sec of sound data recorded from a 3-

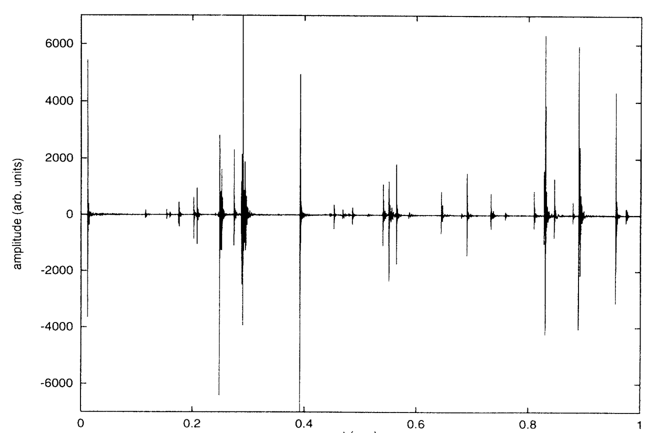

FIG. 3. Amplitude vs time for 1 sec of sound data from a sheet 3 mil thick.

mil sheet. The amplitude $i(t)$ is proportional to pressure, so the energy in the signal is $E(t) \sim i^2(t)$. The sound is clearly resolved into many discrete *clicks*. The integrated energy of the clicks ranges over at least six orders of magnitude. Figure 4 shows a single large click from the same run. This click appears to be a superposition of several distinct vibrational modes.

The source of the sound is evident if one examines the sheet while straining it. Each click is emitted during a sudden change in the rough conformation. The visible change is often localized to a region of size ξ, and can be ascribed to the buckling of a single facet bounded by ridges. The largest clicks originate from the buckling of regions much larger than the roughness length scale. They can probably be interpreted as the superposition of many smaller events. This may be the origin of the internal structure seen in Fig. 4. Some clicks demonstrate a discrete memory effect. By alternately stretching and compressing the sheet, a single buckling event can be induced and reversed, emitting a click in both the forward and reverse portions of the cycle. Under favorable circumstances, this cycle can be repeated dozens of times.

These observations support a simple picture for the mechanism of sound generation in the disordered sheet. According to the spin-glass analogy, the disordered sheet has many nearly degenerate energy minima corresponding to distinct rough conformations [3]. The work done to strain the sheet is stored as elastic energy. When the total work done is enough to overcome the potential barrier to a neighboring minimum, the sheet buckles and the stored energy is released as heat, vibrations in the material, and the sound of a click. This picture is reminiscent of the Barkausen effect, the magnetization jumps during a field reversal in the random-field Ising model, and acoustic emissions during microfracturing processes [7–9]. These are all examples of externally driven systems with quenched disorder, and it is natural that the sound from a disordered elastic sheet should be included in this category.

The five runs made at fixed gain let us make a comparison of the sound from different thicknesses of Mylar. Because we had limited control of the straining process and because the cross-sheet comparisons rely on absolute measures of the energy, the results in this paragraph should be regarded as qualitative only. A detailed discussion of the data will be avoided. At a constant strain rate the energy in the largest clicks increases with the thickness h like h^2 while the total power in the sound stays roughly constant. This implies that the number of clicks per second decreases like h^{-2}. The upward trend in the energy of individual clicks is probably due to the energy scale Yh^3 associated with the material, where Y is the Young's modulus. The downward trend in the number of clicks can be accounted for by the postulate that the number of clicks emitted at fixed strain rate is proportional to the density of frustrated ridges in the sheet $N_r \sim (L/\xi)^2 \sim (L/h)^2$.

Figure 5 shows the normalized energy autocorrelation function

$$C(t) = \frac{\langle E(t')E(t'+t)\rangle - \langle E\rangle^2}{\langle E^2\rangle - \langle E\rangle^2} \tag{1}$$

for one run each of h = 0.5, 1.0, 3.0, 5.0, and 10.0 mil. Note the cusp at the origin. A log-log plot of the data (inset) demonstrates that the autocorrelation function is not adequately described as an algebraic decay. A log-linear plot (not shown) demonstrates that it is also not an exponential decay. Figure 6 shows that all five curves are well described by a stretched exponential $C(t) \sim \exp[-(t/t_0)^\beta]$. A fit by eye gives $\beta = 0.3$–0.4 and $t_0 = (3 - 5) \times 10^{-2}$ msec. Stretched exponential correlation functions are commonly found for systems with glassy relaxation dynamics [9,10]. Our autocorrelation functions all achieve a minimum in the vicinity of $t \approx 10$ msec, and thereafter exhibit fluctuations of magnitude 0.01. We have confirmed that these late-time ($t \gg t_0$) fluctuations are due to the poor statistics associated with

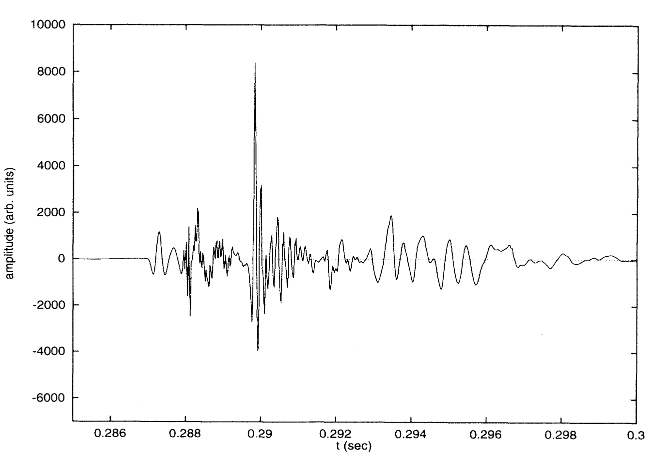

FIG. 4. The large click near $t = 0.3$ sec in Fig. 3.

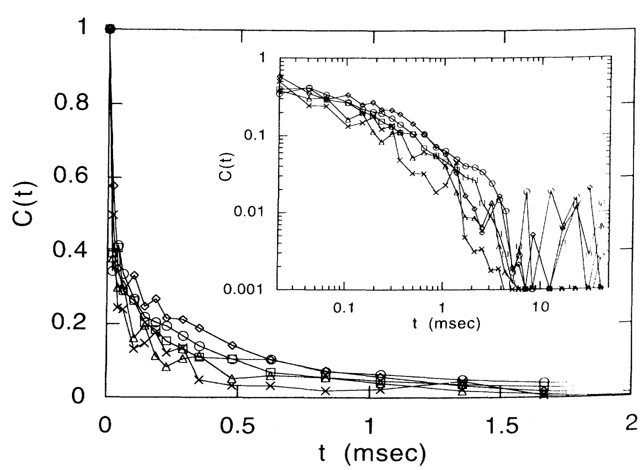

FIG. 5. The energy autocorrelation function for $h = 0.5$ (○), 1.0 (□), 3.0 (◇), 5.0 (△), and 10.0 mil (×). Inset: log-log plot of the same data.

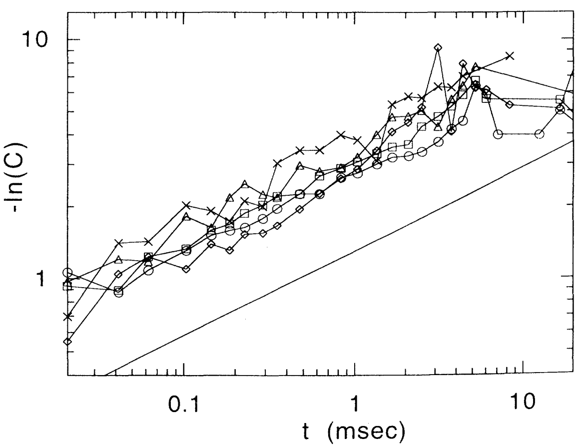

FIG. 6. Plot of $-\ln(C)$ (symbols same as in Fig. 5). The straight line is a stretched exponential with $\beta = 0.35$.

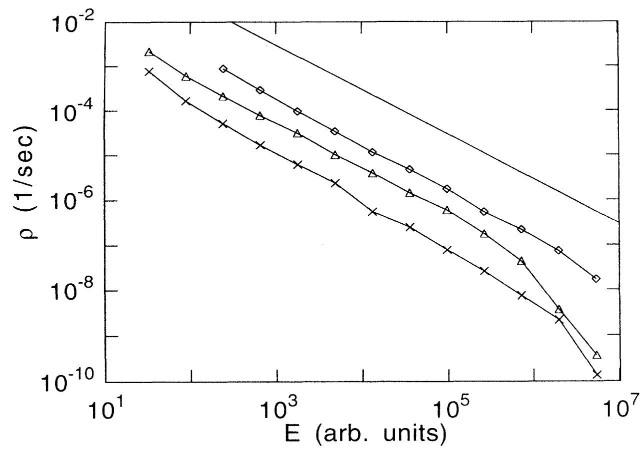

FIG. 7. Plot of the click energy distribution function for h = 3.0 ($\Diamond$), 5.0 ($\triangle$), and 10.0 mil ($\times$). The straight line is a power law with $\alpha = 1.0$.

the largest and least frequent clicks in the sample.

The most interesting characteristic of the sound is $p(E)$, the probability per unit time to encounter a click of energy E. One way to measure this would be to write an algorithm that identifies individual clicks. Unfortunately, we find that the computer identification of smaller clicks is an ambiguous task. Instead, we work with the running sum $E_\Delta(t) \equiv \sum_{t'=t}^{t+\Delta} E(t')$ and define $p(E;\Delta)dE$ as the probability that $E \leq E_\Delta < E + dE$. The estimate for the click energy distribution $p(E) \approx p(E;\Delta)/\Delta$ is valid when (i) $\Delta \gg t_0$, (ii) $\int_0^E dE' E' p(E';\Delta) \ll E$, and (iii) $\int_E^\infty dE' p(E';\Delta) \ll 1$. The first condition ensures that a click is not counted twice. The second condition ensures that the value of E_Δ is dominated by the *largest* click in Δ. The third condition ensures that large clicks are not so common that they hide the majority of smaller ones.

Figure 7 shows our results for $p(E)$ for one run each of thicknesses h=3.0, 5.0, and 10.0 mil. We have chosen $\Delta = 10$ msec to satisfy condition (i). Due to low-amplitude noise in the signal, conditions (ii) and (iii) fail when $p \gtrsim 0.001$. We see that the click energy distribution exhibits a power law $p(E) \sim E^{-\alpha}$ over four decades in the energy. A least-squares fit gives $\alpha = 1.1 - 1.0$. For the 0.5- and 1.0-mil sheets the clicks were too numerous. Conditions (ii) and (iii) were not satisfied and p could not be determined. Although our best results are presented here, we have also examined the sound from crumpled sheets made from materials other than Mylar and with a variety of different sizes. In all cases where p can be determined we find a power-law regime in the distribution of click energies with an exponent ranging from 1.2 to 1.0. We suggest that this result is universal.

Power-law distributions have been measured experimentally in many driven, disordered systems. Studies of the pulse energy distribution in the Barkhausen effect find a power-law regime with an exponent $\alpha = 1.3 - 1.6$ [7]. Comparable studies of acoustic emissions from microfracturing processes and martensitic transformations report $\alpha = 1.3 - 1.5$ and $2.6 - 3.9$, respectively [9,11]. An important distinction is that many of these papers also find a power-law distribution for the duration of the measured pulses. The stretched exponential we report above excludes this possibility for crumpled sheets. A power law in a driven, disordered system is frequently interpreted as an indication of self-organized criticality [12] or as the result of a random multiplicative process [13]. Before we can distinguish between these and other possibilities, future work will have to clarify the mechanism of sound generation.

One motivation for examining the click energy distribution $p(E)$ is to probe the length scales associated with the crumpled sheet. Our initial expectation was that there would be a single energy scale, determined by the roughness length ξ. The result $p(E) \sim 1/E$ implies there is no typical energy scale in the crumpled sheet. This may be an indication of many relevant dynamical length scales in the system. If this is the case, then the upper cutoff of the click energy distribution is determined by the finite size of the sheet and the lower cutoff is determined by h or ξ. However, there is a second possible source of variability in click energy. The distribution of elastic stresses in a rough configuration is likely to be extremely inhomogeneous. The variation in click energies over six orders of magnitude is probably due to both effects.

The most obvious criticism of our experiment is that we strain the sheets by hand, so the reproducibility of our results is in question. Nevertheless, we can borrow an argument from the theory of isotropic turbulence and note that the strains on the sheet are applied at a length scale $l \sim 1$ cm, which is much larger than the length scale ξ at which the strains are relaxed. Similarly, the *longest* time scale measured for the clicks is 10 msec. This is much faster than the macroscopically slow changes in the boundary conditions. We therefore expect that the detailed nature of the driving force is irrelevant for our results. We are supported in this by the robust nature of the click energy distribution $p \sim 1/E$. Experiments under more controlled conditions should resolve this issue.

In conclusion, we have demonstrated that elastic sheets with quenched curvature disorder have a variety of glassy characteristics including many distinct energy minima, discrete memory, and stretched exponential relaxation. The sound produced by these sheets is a direct consequence of the complex energy landscape associated with glassy systems. This paper raises a number of interesting questions. (1) The most pressing experimental issue is the refinement of the experimental procedure and the verification of these results under more controlled circumstances. (2) One would like to better understand the mechanism of click generation, to assess the extent to which the energy in the sound of a click represents the elastic energy released during a buckling event. (3) Lastly, can the theories already developed for elastic sheets with quenched disorder account for the stretched exponential autocorrelation function and the power-law distribution of click energies?

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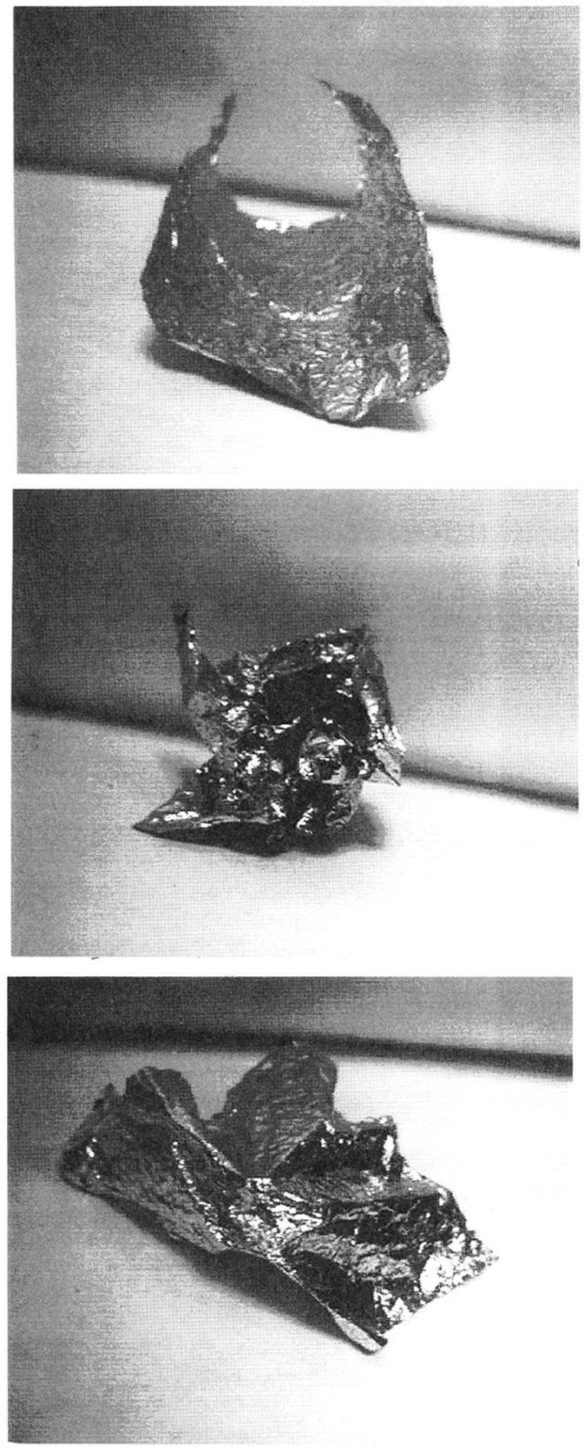

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